Homework 3; Due Wednesday, 09/21/2016

Answer-Only Questions. Credit based solely on the answer.

Question 1. Construct truth tables for the following:

(a) not(A and (not B)),

Solution.

A	B	A and (not B)	not(A and (not B))
T	T		
T	F		
F	T		
F	F		

(b) not(A and B),

(c) (not A) and (not B),

(d) (A and (not B)) or (B and (not A))

Question 2. Negate the following:

(a) A is true and B is false.

(b) A is false or B is true.

(c) A is true and B is true.

(d) A is true or B is true.

Question 3. Construct truth tables for the following:

(a) A or (B and C),

Solution.

A	B	C	B or C	A and (B or C)
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

(b) (A or B) and (not C),

Question 4. List all possible different functions $f : \{a, b, c\} \to \{1, 2\}$

Solution.

x	$f_1(x)$
a	1
b	1
c	1

Medium-Answer Questions. Provide brief justifications for your responses.

Question 5. Which of the following are statements? Explain.

(a) Pythagorus was friendly.

(b) There exists some number x such that $e^x = 1 - x^2$.

(c) The square root of every positive integer is an irrational number.

(d) Aristotle was Greek.

(e) There are an infinite number of prime numbers.

(f) $x^2 - x + 1 = 0$.

(g) Let n be an integer. Then n is either even or odd.

(h) The number $\sqrt{5}$ is rational.

Question 6. Let $F : [2, 1 + e] \to [0, 1]$ be the function defined via the assignment $F(x) = \ln(x - 1)$.

(a) Show that F is a bijection.

(b) Compute the function F^{-1} and state the domain and range of F^{-1}.

Full Proof Questions. Provide complete justifications for your responses.

Question 7. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = ((x^4+1)y,\, x-1)$.

1. Show that f is bijective.
2. Find the inverse function of f, and carefully justify that this is the inverse.

Question 8. Consider the function $D\colon \mathbb{P}_4 \to \mathbb{P}_3$, given by taking the derivative. That is, for a polynomial f, it is defined as $D(f) = \frac{df}{dx}$.

1. Prove that D is surjective.
2. Prove that D is *not* injective.

Question 9. Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = e^{cx}$ for some constant $c \in \mathbb{R}$ (not zero). Prove, using induction, that $f^{(n)}(x) = c^n e^{cx}$. (Note that by $f^{(n)}$, we mean the nth derivative of f.)
You should only use the following facts (it is possible you don't use all of them) in your argument:

(i) $\dfrac{d}{dx}e^x = e^x$

(ii) the chain rule

(iii) the product rule

(iv) if c is a constant, then $\dfrac{d}{dx}c = 0$.

Question 10. Prove that $n + 3 < 5n^2$ for any integer $n \geq 1$.
Hint: *Use induction.*

Question 11. Prove, using induction, that for any $r \in \mathbb{R}$ with $r \neq 1$, and any integer $n \geq 0$,

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}$$